Justifying Claims Related to Similarity - Extension Module
THE FACILITATOR'S COMMENTARY

Session 1

Justifying Claims Module Session 1 Video clip #1

Watch the clip: MVI_0063.MP4

Background Information
This excerpt comes near the beginning of Session 1. Teachers were introduced to terminology used throughout the Justification Module: justification, claim, and key idea. They have completed a task having to do with claims made in a hypothetical mathematics classroom.

The excerpt follows a presentation in which two teachers, who had worked together on the task, presented a justification why Student B's claim was not true. In their explanation, they had highlighted the idea that to prove something true, one example was not sufficient, but to prove something not true, only one counter-example was needed.

The Facilitator's Commentary
At the start of the excerpt, I talk about the difficulty middle grades and early high school students have in understanding what is required to justify a mathematical claim. It was interesting that the teachers' comments included language-based examples that they might use to help their students understand the difference in the evidence needed to justify or prove that a claim is true and the evidence needed to justify that it is not true.

This brief discussion signaled to me the teachers could be flexible in their working with students by connecting argumentation in mathematics to argumentation in other disciplines. This is an important skill for a teacher to help students expand their informal, example-based justifications to more formal, generalized justifications.

Justifying Claims Module Session 1 Video clip #2

Watch the clip: MVI_0065.MP4

Background Information
This excerpt takes place after the teachers have watched Alex's & Justin's clip involving justifying that corresponding angles of dilated triangles were congruent. Alex drew an angle and extended the rays to show that the length did not affect the angle. Justin moved around cutouts of the original triangle angles to show that they

fit—were congruent to the corresponding angles of the dilation image. In the Alex and Justin video, Jennifer, the teacher, had not simply accepted the students' justifications without asking, "Why does that happen?" At one point, she mentioned that Alex "eyeballed" the two congruent corresponding angles and subsequently referred to connecting the justification to having dilated the original triangle.

The Facilitator's Commentary
Since the teachers had watched and debriefed Jennifer's video in their initial LTG professional development, I intended here to emphasize how a teacher might work from students' reasoning in order to build transformation-based justifications. In this discussion, one teacher mentioned that Alex's thinking about the size of the angles was based on extending the angle rays to show that even if they got bigger, the angle stayed the same. And, thus, corresponding angles were congruent.

While it was clear from the video that Alex understood why a change in the ray—or the length of a segment—didn't alter the measure of the angle, I built on what the teacher said about Alex to introduce a common error many middle and some high school students make about angles. After describing the misunderstanding, I asked, "How can you build on the idea Alex is working on?" The teachers' responses indicated to me that they were thinking about angles in terms of rotations and had an understanding of why some students could think otherwise.

I had two purposes in this short conversation: the first was to encourage the teachers' recognition and understanding of the role "key ideas" play in formulating justifications (one of the main mathematical foci), and the second was to call attention to the importance of using students' responses, both correct and incorrect, as opportunities to build key concepts.

Justifying Claims Module Session 1 Video clip #3
Watch the clip: [MVI_0065.MP4]

Background Information
This excerpt shows the discussion further along in the debrief of the Alex and Justin video. The teachers' comments are in response to the question whether Justin "proved" his claim about congruent corresponding angles.

The teachers in this PD group teach mathematics in diverse middle and high schools. The videos in this module portray 6th and 8th grade students. I expected that there would be some differences in the PD teachers' thoughts about Jennifer's use of the term "proved."

The Facilitator's Commentary
During the discussion, I realized how important the matter of proof and proving was to the teachers' practice. As might be expected, the high school teachers by and large felt that "prove" should mean "proof," although one high school teacher accepted the use of "proved" in a 6th grade class such as Justin's. Many of the middle

grades teachers thought that while "proof" should be reserved for formal products—those mathematical statements that are always true—they also thought there could be a less narrow definition of "prove" at the middle school level.

It also became apparent that although the high school teachers preferred that the term "prove" be used only in the context of creating a proof, not all of their students understood the demands of a proof. Both high and middle school teachers spoke about using alternate terms, such as show, give evidence, and explain as a way of helping their students gain experience in creating mathematically-based justifications, and thereby moving them to create more generalized justifications.

The dilemma the teachers expressed—how to work with students not yet ready to create formal proofs—stems, in part, from the yearly tests their students must take. As the PD facilitator I had a challenge. How do I incorporate consideration for this reality facing the teachers? How do I conduct training in teaching using methods developmentally appropriate for their students and not sacrifice mathematical rigor?

The question I asked near the end of the excerpt about justifying the preservation of corresponding angles in a translated image was asked to bring the discussion back to considering the power of teaching geometry from a transformations-based foundation.

Session 2

Justifying Claims Module Session 2 Video clip #1

Watch the clip: MVI_0067.MP4

Background Information

Teachers had completed the task on similar triangles in preparation for viewing a classroom video. The excerpt comes from the debrief of their problem solving.

The Facilitator's Commentary

I started the debrief with the question, "What key ideas did you draw on in your justification?" I knew the task itself was not a true problem for the teachers, but the discussion also had to address which key ideas contributed in making a more generalized justification about similar rectangles. The goal in this debrief was to heighten teachers' awareness of why they chose particular key ideas to justify the problem's claim.

After the first teacher to speak explained that he used the key idea of equivalent within-figure ratios to justify similarity, I asked a question intended to heighten his, and others', awareness of why he chose that key idea. The question prompted a discussion focused on the difference between a statement that could be applied to all rectangles similar to the given set vs. a statement that applied to pairs of rectangles similar to the given set.

In the second part of the excerpt, a teacher offered a different key idea, based on dilating the original unit of measure and redefining what was called 1 unit. I saw this as a good opportunity to extend the teachers' thinking about the relationship between generalizations about similarity and generalizations about measurement.

Justifying Claims Module Session 2 Video clip #2

Watch the clip: MVI_0068.MP4

Background Information

This excerpt comes after the teacher's have watched Makayla's and Victoria's clip. They had already seen this clip during the summer training. In the debrief of that viewing, we concentrated on characterizing students' thinking. In the current debrief, the teachers were to focus on students' justifications of why a 2 x 3 rectangle would not belong in the set of similar rectangles pictured.

The Facilitator's Commentary

Several interesting comments emerged from this small group discussion. The teachers recognized there was a distinction between observations students made and their justifications for deciding which rectangle did not belong in the set. For example, they noted that Makayala could describe how the dimensions changed as the similar rectangles got larger and that she made a list of the dimensions saying there was a pattern. But they did not think Makayla understood why or how that pattern was related to similarity.This distinction is important to recognize. Students who cannot go beyond describing a pattern may not yet be ready to create more generalized justifications.

Justifying Claims Module Session 2 Video clip #3

Watch the clip: MVI_0068.MP4

Background Information

During the discussion about the Makayla and Victoria clip, the teachers had addressed a question about the differences between what students understand and what they can justify. Responses raised the significance of students' ability to identify a pattern recursively and explicitly, and which way might be more useful in developing a generalized justification.

The Facilitator's Commentary

In this excerpt, we have turned to the matter of what a teacher might do to move a student such as Makayla toward recognizing a more generalized pattern and how that pattern relates to all the rectangles similar to the given set.

The two suggestions teachers made in this excerpt get at slightly different approaches. The first would challenge a student to consider rectangles with larger dimensions, such as a length of 50 units, making it more tedious for her to continue *to add recursively; however, she could arrive at a solution by continuing to add recursively if she needed to. The second suggestion uses the student's own way of thinking, that is, adding a certain amount to one side and doubling the amount added to the other side. However, the given dimension would be one that is not easily doubled or halved, such as 3 units or ½ unit.*

Since we had been discussing this video for a long time, I decided not to ask any more questions about it. But if we had had the time and I didn't think the teachers needed to move on, I would have asked a question about the opportunities each strategy afforded a student to gain new understanding.