Definitions of Similarity - Extension Module
THE FACILITATOR'S COMMENTARY

Session 1

Introduction to the video clips
The first three clips should be viewed as a series of excerpts that follow a progression of PD activities intended to emphasize the role of definitions in developing a conceptual understanding of similarity based on geometric transformations. The clip excerpts portray (1) setting up a problem, (2) a discussion of approaches to teachers' problem solutions, and (3) a discussion of implications for pedagogy. The fourth clip relates to another problem that centers mathematical definitions and the challenges students might have with this problem.

Definitions Module Session 1 Video clip #1

Watch the clip: MVI_0057_Clip 1

Background Information
Before video clip #1, teachers were given the following prompts:

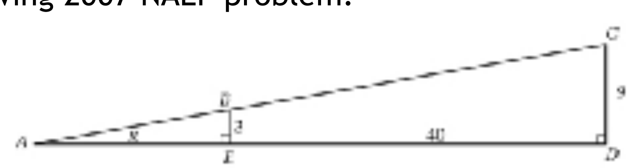
- *What are some formal and informal definitions of similarity?*
- *Which of these definitions do you expect are already familiar to your students?*

In recording teachers' responses, the facilitator asked them about the source of their definition. Teachers' responses included the following definitions and sources:

- Congruent corresponding angles and proportional corresponding sides
- Same shape, different size
- Two figures that have undergone a sequence of rigid motions/non-rigid motions
- There exists a dilation of some scale factor *K*

The Facilitator's Commentary
Teachers had worked on the task below, with is an 8th Grade NAEP item.

Solve the following 2007 NAEP problem.



16. The figure above shows two right angles. The length of AE is x and the length of DE is 40.

Show all of the steps that lead to finding the value of x. Your last step should give the value of x.

I anticipated that the mathematics would not be problematic for the teachers. Since teachers had experienced previous solution discussions where a variety of methods was emphasized, I hoped that teachers would use multiple methods to solve this problem. So after suggesting that they solve the problem individually and then share with others, I added to the task by asking teachers to look for connections between the approach they used in solving the problem and the list of definitions we collected from the earlier prompts.

My goal was to highlight the role of definitions and how they are related to solutions. Definitions are often presented and discussed in isolation before students have seen any examples of the concept in question or solved any problems related to that concept. Definitions are connected to our strategies for solving a problem. The language of a definition leads to problem interpretation, to how accessible the problem is, and to self-monitoring while working on a solution.

Definitions Module Session 1 Video clip #2

Watch the clip: MVI_0057_Clip 2

The Facilitator's Commentary

Teachers gave several different approaches to solving the problem.

- *-Set up a proportion*
- *-Do a dilation and then check algebraically*
- *-Find the scale factor for triangle ACD*
- *-Use the diagram to visualize successive dilations: a 1-dilation, a 2-dilation, and a 3-dilation, interpreting the lengths of corresponding sides as 3, 6, 9, and x, 2x, 3x.*

With each explanation of the problem-solving approach used, I was looking for comments teachers made relating the definition of similarity from the list given earlier to the method used to solve the problem. My intent was for teachers to reflect on the problem and their solution method in order to go beyond the most commonly used definition and solution approach.

The majority of the teachers used a proportion to solve the problem, as is typical. They knew that the goal of the PD was to go beyond using "automatic" procedures and gain experience with using a transformations-based approach to teaching similarity, but I didn't want them to feel apologetic about having used the procedure they and others have typically used. For these teachers, this was not truly a problem, and so setting up a proportion and solving it algebraically was an automatic response to the task.

Throughout the discussion I tried to keep my comments to a minimum, or to not comment at all immediately after a teacher spoke. I judged that a good pattern at this point in the discussion would be for teachers to extend, offer a contrast or *clarification, or comment on other teachers' responses rather than if I reacted first. The teachers' comments flowed from teacher to teacher, demonstrating their awareness of the difference between performing a procedure and thinking through the problem in order to understand why a certain procedure or strategy for solving the problem made sense.*

The teachers' developing awareness of the connection between definitions and approaches to solving a problem is illustrated by the discussion of the dilemma teachers face when students lack particular skills needed to solve the problem in a "traditional" way. One teacher responded to this dilemma with an account of her student who did not know (formal) algebra, yet could use her understanding of the diagram and her intuition to work through the problem. In this case, definitions related to scale factor and dilations implied multiple approaches to solving a problem.

In reflecting on the teachers' comments in this video clip, I did note that even the teachers who used transformations-based methods to solve the problem, "checked" their answers with a method they felt more accustomed to.

This video clip illustrates the ongoing development (the evolution) of the teachers' awareness of a connection between definitions and approaches to solving a problem. The clip portrays teachers coming to recognize that definitions are not only connected to mathematical properties but also to mathematical actions.

Definitions Module Session 1 Video clip #3

Watch the clip: MVI_0057_Clip 3

The Facilitator's Commentary

In this video clip, allowing teachers to consider an intriguing statistic—the low percentage of students who solved a problem correctly—led to a rich discussion of approaches to problem solving, analysis of student misunderstanding, and an exploration of ways to help students with their difficulties.

The most common error students made on this task was setting up the proportion incorrectly. The teachers tried to make sense of why students had trouble. There seemed to be a consensus that students misunderstood the diagram and that instead of seeing two overlapping triangles, the students saw only one. The teachers recognized that the nature of the triangles used in the diagram made it impossible for students simply "plug in" numbers and come up with a correct equation to solve the problem.

When the discussion turned to ways to help students past this stumbling block, one suggested strategy involved changing the diagram so that students saw two distinct triangles. Other suggestions involved strategies that would help students build greater understanding of the multiple ways to define similarity. Giving students *experience with dilations before presenting a problem such as the NAEP item might lead students to define similarity in terms of a transformation and thus interpret the diagram as a dilation. Giving students experience with measuring overlapping angles supports the definition of similar triangles as those that have congruent corresponding angles.*

The first suggestion about separating the two triangles in the diagram was not as directly related to a transformation-based definition of similarity as the ones that followed, but it could be if that were the teacher's goal. Although my goal was to emphasize a transformational approach to similarity, I did not comment immediately to redirect the discussion. I have found that with patience and careful prompting, having students—in this case, the teachers—give the key ideas has a greater impact than if I had done it. In this clip, allowing the teachers to continue the discussion without my comments may have lead to the suggestions for using a transformational-based approach to teaching and learning about similarity.

Definitions Module Session 1 Video clip #4

Watch the clip: MVI_0057_Clip4

Background Information
Teachers have a task to apply several similarity definitions to pairs of shapes and assess how students would determine whether a given pair were similar or not using a particular definition. For example, given two congruent triangles, what would students say about their similarity using the definition "same shape, different size"?

The Facilitator's Commentary
The video clip focuses on a pair of non-congruent circles placed side-by-side. The teachers realize that the problem for students is that there are no congruent angles or proportional sides. One teacher raises the question about students' ability to find the center of each circle so they can do a transformation, maybe a translation, and determine whether the circles are similar. (Here we're assuming students have not yet discovered that all circles are similar.)

The teachers in this group were from high schools and middle schools, but few had used transformations in conjunction with any mathematics topics that were not included in the Standards. At this point, I wanted the teachers to go beyond considering a transformations-based approach for congruence and similarity only. So I asked a question about how a sixth grader might find the center of a circle. When one of the teachers talked about folding the circle to find the diameter, I expanded on that, saying that you needed to fold it twice. I ended with a comment about the power of using transformations in many mathematical problem contexts.

Session 2

Definitions Module Session 2 Video clip #1

Watch the clip: MVI_0060.MP4

Background Information

This video excerpt follows a debriefing of teachers' solutions to the Grade 12 NAEP problem from 1992. After sharing their solution strategies, teachers discussed what students might stumble on. The given dimensions on the two triangles begged a solution method of setting up a proportion to solve for *x*. However, the teachers also realized that many students might not identify the corresponding sides correctly. I asked the teachers to think about what they as teachers might do to help such students successfully solve the problem.

The Facilitator's Commentary

The discussion that followed gave me some insight, as the facilitator, into the range of teacher thinking about the use of a transformations-based approach to teach similarity. I looked for the teachers' suggestions for helping students as well as their comments on suggestions of others. For example, when one teacher talked about naming and labeling corresponding angles, thus cuing a student as to which angles were corresponding, the next teacher responded that doing so might defeat the intent of the problem.

The teacher who said she was trying to "avoid the static" approach also saw a potential stumbling block for students using transformations. That is, if students didn't notice that the arcs on the diagram identified two congruent triangles, they might try to reflect one of the triangles over a line drawn through the common vertex, parallel to the two bases. At the end of this excerpt, one teacher suggested giving students experience with transformations before solving problems such as the NAEP problem.

In this excerpt, I tried to stay out of the discussion as much as possible. The flow of the discussion indicated to me that teachers were beginning to independently consider the principles we had been focusing on to address pedagogical situations and to reflect on their own practice.

Definitions Module Session 2 Video clip #2

Watch the clip: MV1_0061.MP4

Background Information

Just before this excerpt, teachers had had a discussion of Terri's clip 1. In the discussion, teachers noted that Terri used two quite different examples of similar figures in her diagrams: two rectangles with grid marks placed side-by-side and two non-isosceles trapezoids with vertices labeled and with one rotated and oriented differently from the other. After speculating a little about what Terri's goals might have been in choosing these diagrams, I decided to move the discussion on to a consideration of how each pair of similar figures could support the development of a transformations-based definition of corresponding sides.

The Facilitator's Commentary
Teachers noticed differences between the two pairs. For example, with the pair of trapezoids there was nothing to count, one trapezoid was turned, and the vertices were labeled. But no one talked about how those features could be connected to helping students develop a more mathematically robust definition of corresponding sides based on transformations.

At this point, I decided to suggest a key question teachers might ask students who were having difficulty determining if the figures were similar without dimensions and with the figures not facing the same way, What could you do to check it out? The hope here is that this question will lead some students to move the figures around. Doing so would allow them to test for congruent corresponding angles, re-orient the trapezoids to find parallel corresponding sides, and discover other properties that are characteristic of similar figures.

The importance of calling attention to the figures—what a teacher chooses to use for examples, diagrams, how questions are posed—all make a difference in students' opportunities to learn the material as intended.

I chose to do a summary of the teachers' discussion, with some interjection from the teachers, because I felt they were beginning to recognize the power of transformations to support teaching. Summaries generally include a synopsis of teachers' comments related to the key mathematical ideas that drive the discussion. They may also include unresolved questions or differences of understandings, approaches, etc. And they may include prompts for things I want students to consider for the next session.

Definitions Module Session 2 Video clip #3
Watch the clip: MVI_0062.MP4

Background Information
This video excerpt comes from the teachers' debriefing of Bree's clip. Before watching the video, the teachers completed and discussed Bree's task.

Form a triangle CCC. Using a ruler, measure its side lengths. Then move one or more of the angles in order to form a triangle CCC which is a different size than the first CCC. Measure the side lengths of this new CCC. Repeat this process a few times. What do you notice?

The Facilitator's Commentary
Just before this video excerpt, I had asked teachers to comment on Bree's students' observations in the class discussion. One teacher mentioned the student's observation

that the triangle could be any size. Since this understanding is key to developing a definition of similarity that includes all possible figures in a similarity class, I wanted the teachers to think about what might have prompted such student observations.

One teacher mentioned that the nature of the task allowed students to create more triangles by merely sliding the given angles around. They were not asked to create a number of similar triangles. Nor did they have to be able to measure the lengths of corresponding sides to determine similarity. Bree's task led the students to use motion (not specifically a transformation) to create additional triangles. It was important that the teachers recognize that Bree's task was more open-ended than others they had experienced, that it led the students to collectively create a set of similar triangles.

A key point of this Definitions extension module session is to explore how to develop transformations-based mathematical definitions in classroom lessons. My intent in this brief excerpt was to highlight the role of the task—its content, the prompts used, and the materials—in developing such a definition of similarity. With Bree's task students actually had to move transparencies to create additional triangles. They were not restricted to constructing side lengths they could easily measure. Thus, students could create infinite similar triangles, a key part of a transformation-based definition.