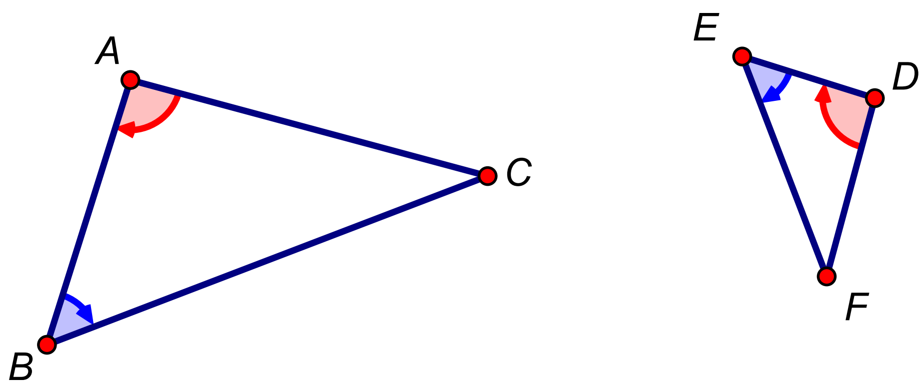
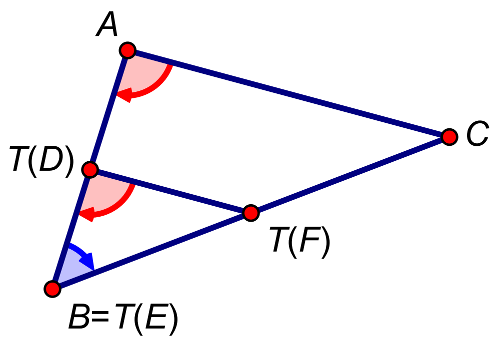
Visual Proof Answer Key: AA Similarity Theorem

Given two triangles △ABC and △DEF such that ∠A ≅ ∠D and ∠B ≅ ∠E. Prove that △ABC is similar to △DEF.



Proof:

By definition of congruence, since∠B ≅ ∠E, there exists a sequence of translations, rotations, and reflections that sends ∠B to ∠E.
There is a choice of where the point D is sent. Since an angle, *by definition*, is two rays with a common endpoint, D must be sent to one of the two rays connected to B: Either ray BA or ray BC. Choose a transformation T that sends D to the ray BA. Since F is on the other ray of ∠E, T(F) will be on ray BC. Here is the picture: (note it is possible that T(D) and T(F) are on the outside of ABC, this is fine, the rays continue to infinity).

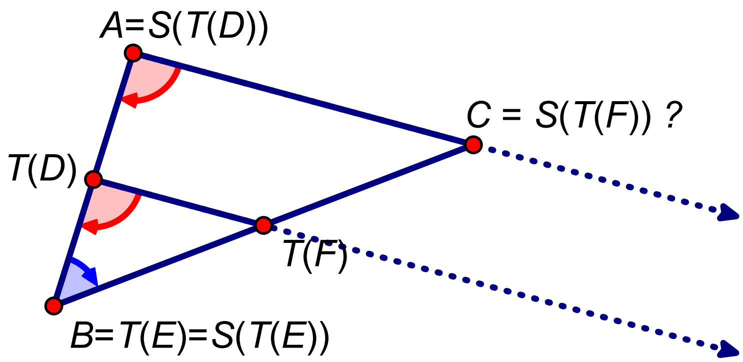


Now let S be a dilation with center B and scale factor |BA|/|BT(D)|.

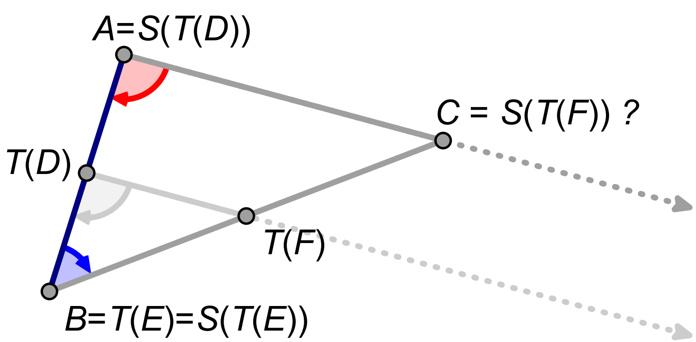
By definition of dilation, the point T(D) is sent by S to point A.

Dilations preserve angles, so that implies that ray T(D)T(F) is sent to ray AC.

We do not know that S sends T(F) to point C. We still need to prove that.
This look like:



Notice that ∠B ≅ ∠S(T(E)) and ∠A ≅ ∠S(T(D)) and segment AB ≅ segment S(T(E))S(T(D)). To weed through all the letters, here is the picture with the relevant parts in color with the rest grayed out:



Here we see that we have two triangles △ABC and △S(T(D)S(T(E)S(T(F)) that have two angles congruent and the contained side congruent.
Therefore, by Angle-Side-Angle Congruence Theorem, △ABC≅△S(T(D)S(T(E)S(T(F))

Unpacking what we have done, and noting that △S(T(D)S(T(E)S(T(F)) = S(T(△DEF)), we see that :

There is a sequence of congruence transformation and dilations that map △DEF to △ABC. Therefore, *by definition of similarity*, △ABC is similar to △DEF.